Volumes of Revolution Calculus of a Single Variable X. Du

- Volume
  - Integrate an area (integrate a two dimensional object yields a three dimensional object)
  - Think about it conceptually: If you take a potato and slice it into very, very thin pieces, you can add the volumes of those potato slices to find the volume of the whole potato. Because the potato slices are very, very thin, linear approximation is a good approximation, and we can simply treat each slice as a very thin cylinder.
  - This can be done with any closed two dimensional figure (not just necessarily circular ones that we will explore below).
- **Disk Method**:
  - Integrate the area of a disk through a variable.
  - If $f(x)$ is rotated about the x-axis, then $V = \pi\int_a^b [f(x)]^2 dx$,where the lower and upper bound are *a* and *b* respectively.
  - Think of $f(x)$ as the radius of your infinitely thin slice and *dx* as the infinitely small height of your infinitely thin slice.
  - More generally, $V = \pi\int_a^b [R(x)]^2 dx$ for a horizontal axis of rotation. $R(x)$ is the radius function of the figure. For a vertical axis of rotation, $V = \pi\int_a^b [R(y)]^2 dy$.
- **Washer Method**:
  - Integrate a doughnut through a variable.
  - Think of this as a disk method except with an inner part removed.
  - If $f(x)$ is farther from the axis of rotation than is $g(x)$, and the area between the two curves is rotated about the x-axis, then $V = \pi\int_a^b \left([f(x)]^2 - [g(x)]^2\right)dx$.
  - In more general terms, $V = \pi\int_a^b \left([R(x)]^2 - [r(x)]^2\right)dx$, where $R(x)$ is the function of the outer radius and $r(x)$ is the function of the inner radius for a horizontal axis of rotation. $V = \pi\int_a^b \left([R(y)]^2 - [r(y)]^2\right)dy$ for a vertical axis of rotation.
- **Shell Method**
  - Integrate the outer surface of a cylinder through a variable.
  - $V = 2\pi\int_a^b \left(r(x)h(x)\right)dx$, where $r(x)$ is your radius function and $h(x)$ is your height function, and the axis of rotation is horizontal. $V = 2\pi\int_a^b \left(r(y)h(y)\right)dy$ for a vertical axis of rotation.